From reversible to irreversible computations

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Structure of Talk

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Introduction.
Reversible Computation.
Irreversible Computation.
The Three Laws (of Equivalence).
Using the Three Laws.
Conclusions and Further Work.

Introduction

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- We'd like to model irreversible computations as a derived notion from reversible computations.

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there is an inverse, $\psi^{-1} \in \mathbf{FxC}^{\mathrm{R}}(b, a)$.
- Such that ψ, ψ^{-1} are an isomorphism.
- We assume the groupoid is strict, so we can denote:
$\mathbf{FxC}^{\mathrm{R}}\, a = \mathbf{FxC}^{\mathrm{R}}(a, a)$.

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- The $\otimes$ operation corresponds to parallel composition.
- We can characterise the morphisms, i.e. circuits, in $\mathbf{FxC}^{\mathrm{R}}a$ inductively.

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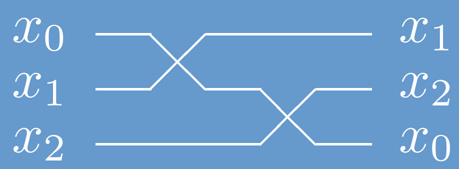

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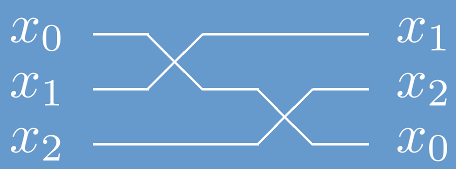
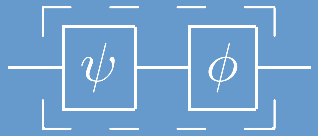


would have $\phi(0) = 2$, $\phi(1) = 0$, and $\phi(2) = 1$.
- The identity, id_a , is a special case of wires.

Reversible Computation...

- - sequential composition - given $\psi, \phi \in \mathbf{FxC}^{\mathrm{R}} a$ we construct $\phi \circ \psi \in \mathbf{FxC}^{\mathrm{R}} a$.



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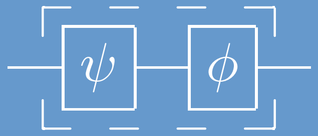

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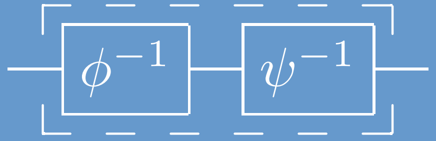

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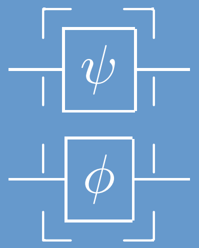
- - parallel composition - given $\psi \in \mathbf{FxC}^{\mathrm{R}} a$ and $\phi \in \mathbf{FxC}^{\mathrm{R}} b$ we can construct $\psi \otimes \phi \in \mathbf{FxC}^{\mathrm{R}}(a \otimes b)$.



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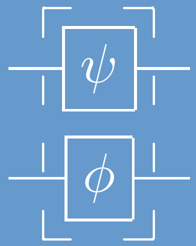

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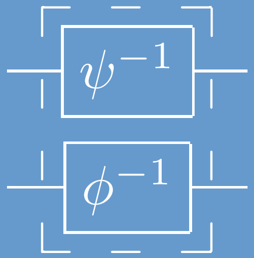

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  i.e. $\neg \in \mathbf{FxC}^{\mathrm{R}}1$ with $\neg^{-1} = \neg$.
- In the quantum case this could be any single qubit rotation.
  i.e. any unitary operation in $U(2)$.

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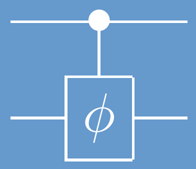

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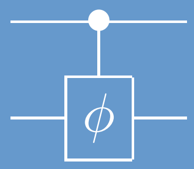

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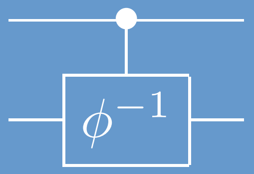

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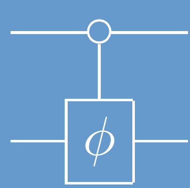

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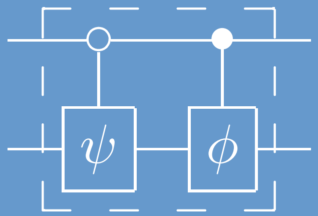

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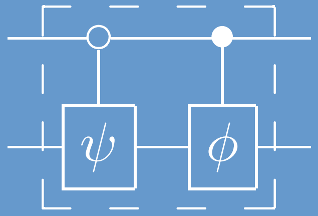

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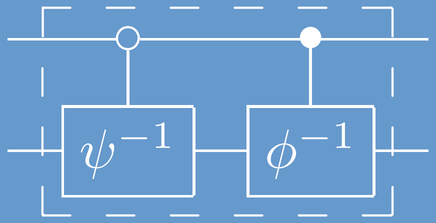

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- We introduce extra equalities for the conditionals:
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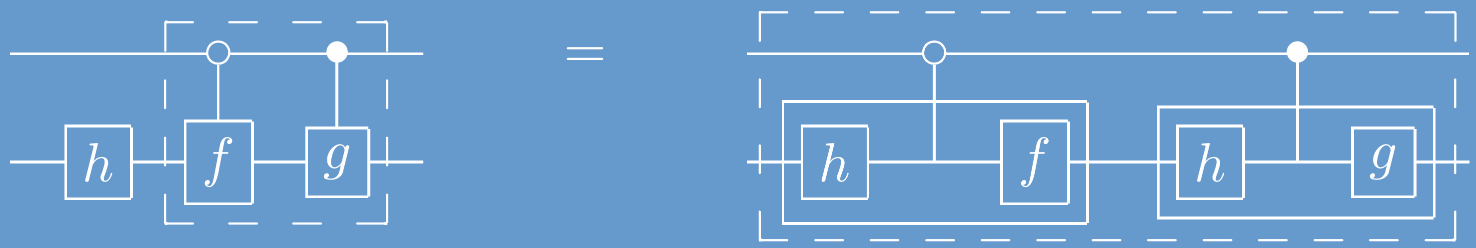

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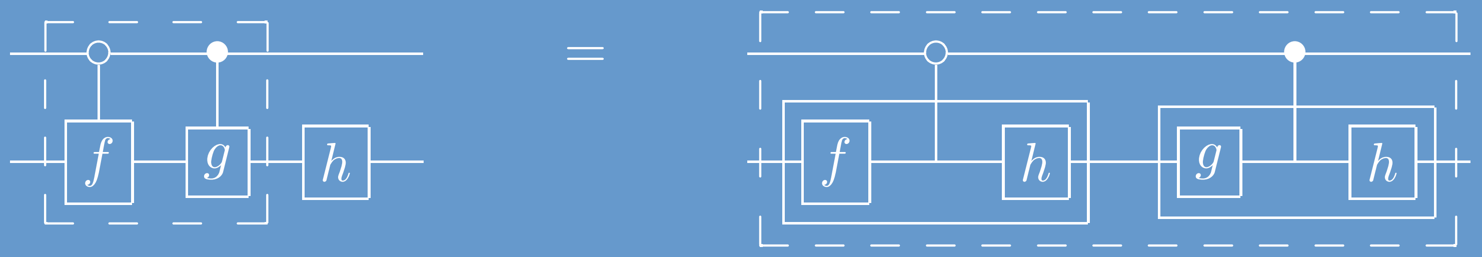

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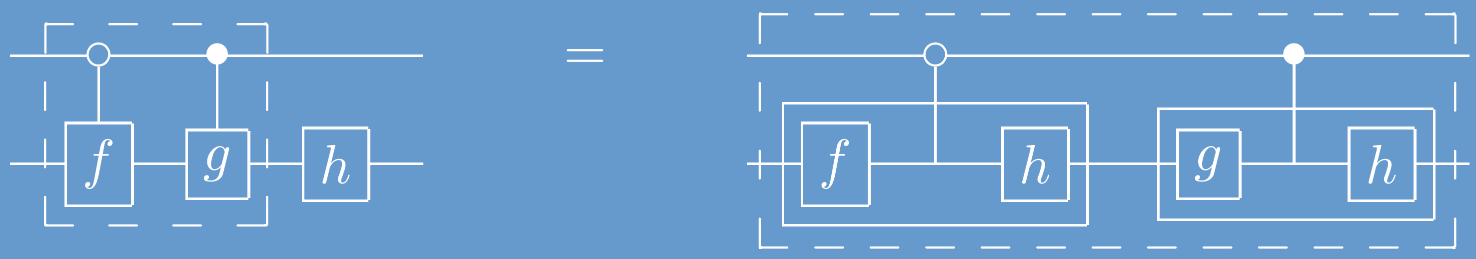

- For $f, f', g, g' \in \mathbf{FxC}^{\mathrm{R}}a$ that $(f \mid g) \circ (f' \mid g') = (f \circ f') \mid (g \circ g')$.

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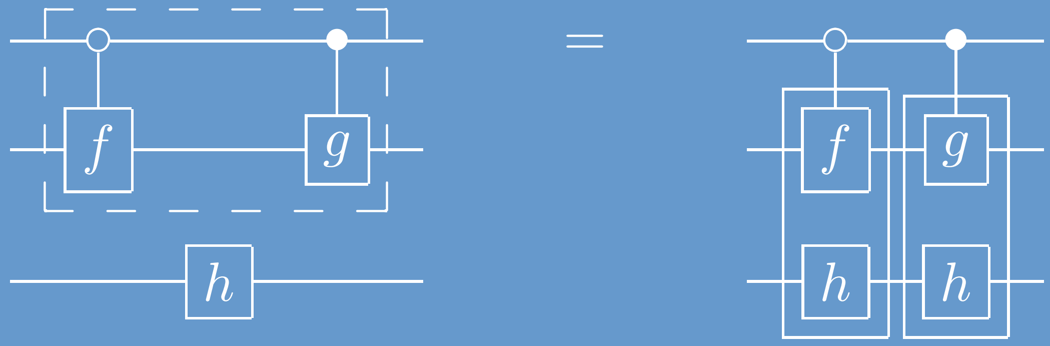

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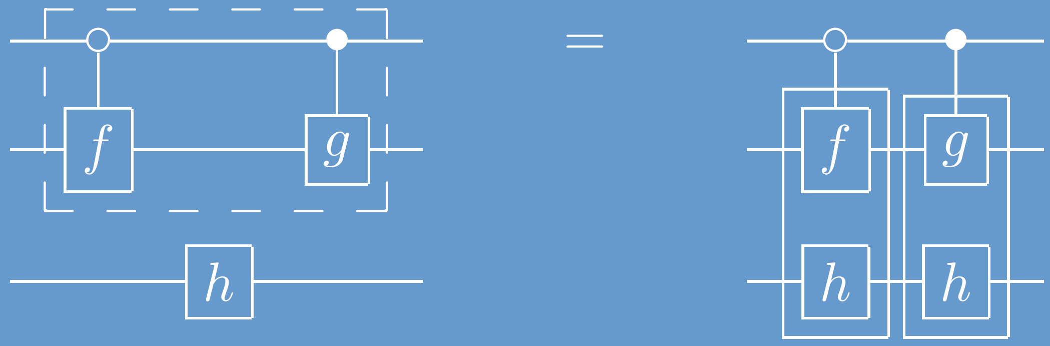

- It has been suggested that we can now simplify the first two to be that $(h \mid h) = (id_1 \otimes h)$.

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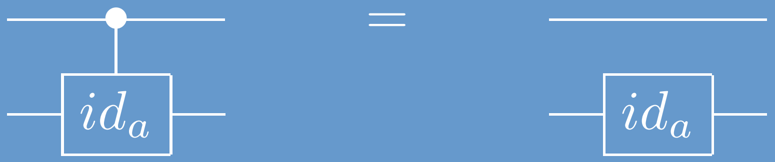

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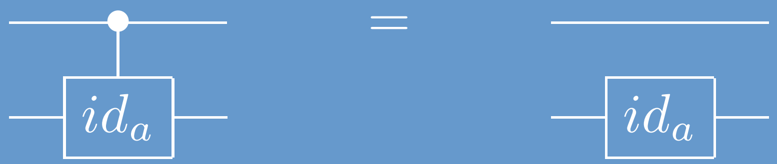
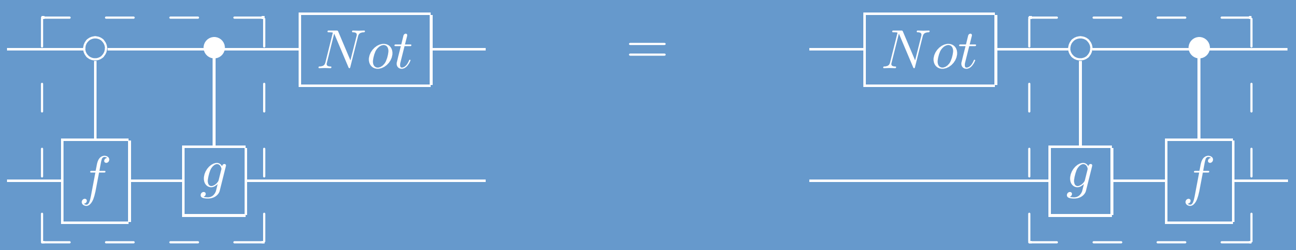


- Moreover, we have for $f, g \in \mathbf{FxC}^{\mathrm{R}} a$ that $(\neg \otimes id_a) \circ (f \mid g) = (g \mid f) \circ (\neg \otimes id_a)$.



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- Note that $\mathbf{FCC}^{\mathrm{R}} \hookrightarrow \mathbf{FQC}^{\mathrm{R}}$, and preserves extensional equality.

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- A bipermutative category is a symmetric bimonoidal categeory where all isomorphisms apart from $c^{\oplus} \in A \oplus B \simeq B \oplus A$ and $c^{\otimes} \in A \otimes B \simeq B \otimes A$ are identities.
- Our models for $\mathbf{FCC}^{\mathrm{R}}$ and $\mathbf{FQC}^{\mathrm{R}}$ give rise to bipermutative categories, where $\mathbb{N}_2 = I \oplus I$ and all the laws stated above hold in all bipermutative categories.

Bipermutative Categories

- There are still a number of coherence laws to be satisfied such as:

$$\begin{array}{ccc} A \otimes (B \oplus C) & = & (A \otimes B) \oplus (A \otimes C) \\ \Big\downarrow {\scriptstyle A \otimes c^{\oplus}} & & \Big\downarrow {\scriptstyle c^{\oplus}} \\ A \otimes (C \oplus B) & = & (A \otimes C) \oplus (A \otimes B) \end{array}$$

and

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- - ψ - is the underlying reversible computation.

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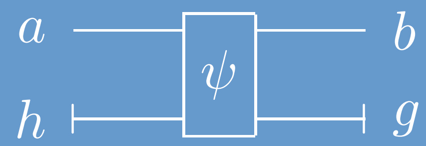
- A morphism in $\mathbf{FxC}^{\mathrm{Ir}}(a, b)$ can be given as a morphism in $\mathbf{FxC}^{\mathrm{R}}((a \otimes h), (b \otimes g))$ with the requirement that $(a \otimes h) = (b \otimes g)$.



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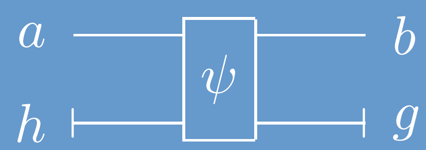


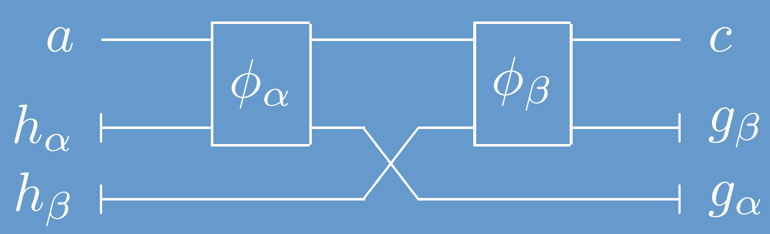
- For any $\psi \in \mathbf{FxC}^{\mathrm{R}}a$ there is an equivalent circuit $\widehat{\psi} \in \mathbf{FxC}^{\mathrm{Ir}}(a,a)$.

$$\frac{\psi \in \mathbf{FxC}^{\mathrm{R}}a}{\widehat{\psi} \in \mathbf{FxC}^{\mathrm{Ir}}(a,a)}$$

such that $\widehat{\psi} = (0, 0, \psi)$, i.e. there is no heap or garbage.

Compositionality

- Given $\alpha = (h_\alpha, g_\alpha, \phi_\alpha) \in \mathbf{FxC}^{\mathrm{Ir}}(a,b)$ and $\beta = (h_\beta, g_\beta, \phi_\beta) \in \mathbf{FxC}^{\mathrm{Ir}}(b,c)$, we define $\beta \circ \alpha \in \mathbf{FxC}^{\mathrm{Ir}}(a,c)$, as



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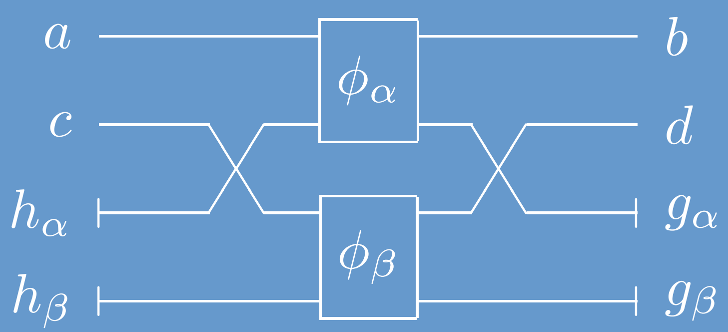

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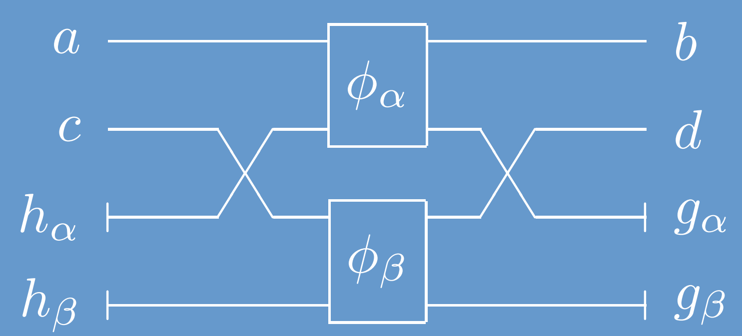

- The neutral element of the tensor, i.e. the empty circuit, can be obtained by lifting $I^{\mathbf{FxC}^{\mathrm{Ir}}} = \widehat{I^{\mathbf{FxC}^{\mathrm{R}}}}$.

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- \- **FQC** - The category of finite quantum computations.
- Morphisms are interpreted as superoperators: $(h, g, \phi) \in \mathbf{FQC}(a, b)$ is interpreted as $\mathrm{tr}_g \circ [\![\phi]\!] \circ 0^h \otimes - \in \mathbf{Super}(a, b)$, where $[\![\phi]\!] \in \mathbf{Super}(h \otimes a, g \otimes b)$ is the superoperator associated to the unitary operator given by interpreting the reversible circuit ϕ . $0^h \otimes - \in \mathbf{Super}(a, a \otimes h)$ initialises the heap and $\mathrm{tr}_g \in \mathbf{Super}(g \otimes b, b)$ is a partial trace which traces out the garbage.

Equivalence

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- For example, the following equivalence holds in **FCC** but not in **FQC**.

Equivalence.

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- How can we characterise the equivalences which should always hold?

The Three Laws - 1

- - Garbage Collection - If a circuit can be reduced into two smaller circuits such that one part of the circuit only acts on heap inputs and on garbage outputs, then that part of the circuit can be removed.

The Three Laws - 2

- - Uselessness of garbage processing - If a circuit can be reduced into two smaller circuits such that one part of the circuit only has an effect on garbage outputs, then that part can be removed.

The Three Laws - 2

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- or alternately

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- we can now simplify the first law to state that a wire that simply connects the heap to the garbage is equivalent to having nothing.

The Three Laws - 3

- \- Uselessness of heap preprocessing - If a circuit can be reduced into two smaller circuits such that one part of the circuit only has effect on heap inputs, and the effect on the zero vector is the identity, then that part can be removed.
  $if \quad h\vec{0} = \vec{0} \quad then$

The Three Laws - 3

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- Both are in $\mathbf{FQC}^{\mathrm{R}}$, and actually only use elements of $\mathbf{FCC}^{\mathrm{R}}$.

Using the Three Laws.

- The third controlled not is eliminated using the second law:

Using the Three Laws..

- The controlled Not operations preserve the zero vector, so we can eliminate the first one using the third law:

Using the Three Laws...

- Finally the bottom wire can be removed by use of the first law:

Conclusions & Further Work

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Thankyou

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For more info see:
http://sneezy.cs.nott.ac.uk/QML

References

[AG05] Thorsten Altenkirch and Jonathan Grattage. A functional quantum programming language. In *20th Annual IEEE Symposium on Logic in Computer Science*, 2005.

[Lap72] M. Laplaza. Coherence for distributivity. *Lecture Notes in Mathematics*, 281:29–72, 1972.

[Sel05] Peter Selinger. Dagger compact closed categories and completely positive maps. In Peter Selinger, editor, *Proceedings of the 3rd International Workshop on Quantum Programming Languages*, Electronic Notes in Theoretical Computer Science. Elsevier Science, 2005.