Shor in Haskell
The Quantum IO Monad

Trends in Functional Programming
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- Classically, the best known solution is $O(2^{(logN)^{\frac{1}{3}}})$ which for large numbers is computationally infeasible.
- The RSA encryption protocol uses this assumption, and hence could be "broken" by a quantum computer.

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- Quantum teleportation enables the use of quantum key distribution, allowing provably secure communication.
- There are already commercial companies offering quantum crytography products (BB84)...

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- While the design of quantum algorithms can make use of the abstractions available in Haskell.
- I shall now give a brief introduction to both quantum computing and the Quantum IO Monad .

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- Qubits can exist in a super-position of both states simultaneously.

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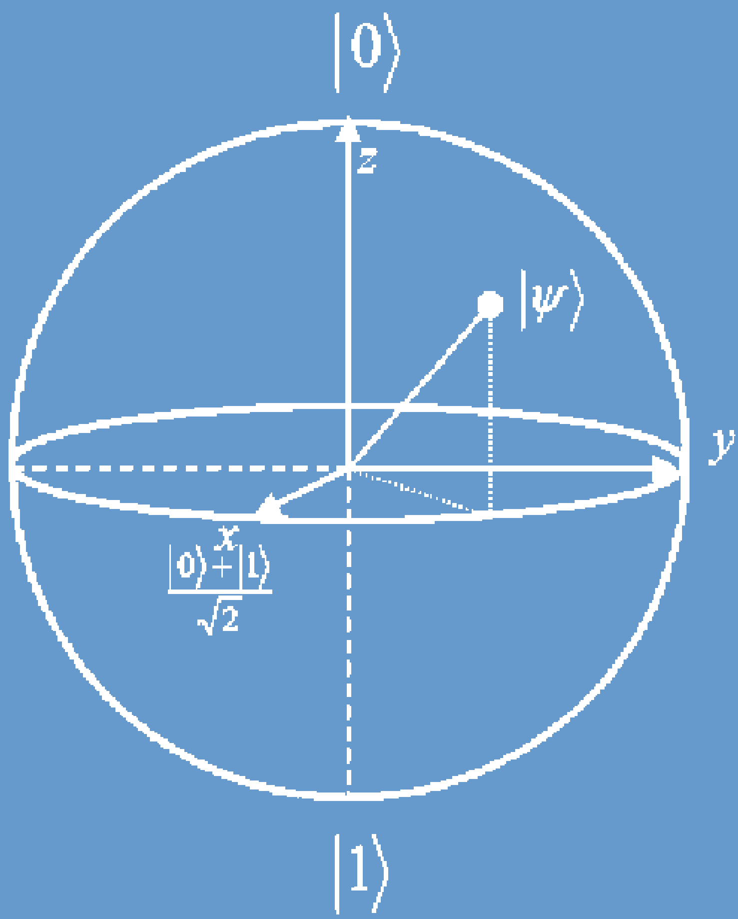
- and with the side condition that $|\alpha|^2 + |\beta|^2 = 1$.
- The Bloch sphere can be used to visualise this...

Bloch Sphere



An arbitrary (single qubit) state can be thought of as any point on the surface of the sphere.

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- Rotations are used in QIO to create the single qubit super-positions.

Qubit Rotations

- Rotations are defined by unitary 2 by 2 complex valued matrices, e.g.

$$unot = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ uhad = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

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$$\mathbf{type}\ Rotation = ((Bool, Bool) \rightarrow \mathbb{C})$$

- which is extended to a member of the U type by

$$rot :: Qbit \rightarrow Rotation \rightarrow U$$

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  which is an equal super-position of the single qubit base states.
- $$\begin{aligned} &|+\rangle :: QIO\ Qbit \\ &|+\rangle = \mathbf{do}\ q \leftarrow |0\rangle \\ &\qquad\qquad applyU\ (uhad\ q) \\ &\qquad\qquad return\ q \end{aligned}$$

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- $$\begin{aligned} &randomBool :: QIO\ Bool \\ &randomBool = \textbf{do}\ q \leftarrow |+\rangle \\ &\qquad\qquad\qquad\qquad c \leftarrow measQbit \\ &\qquad\qquad\qquad\qquad return\ c \end{aligned}$$

Multiple Qubits...

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- For an n-qubit system, the state can occupy a super-position of any of the 2^n bit strings of length n.
- We still have the side-condition that the sum of the squared amplitudes must equal 1 .

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$$swap :: Qbit \rightarrow Qbit \rightarrow U$$

- and create conditional operations

$$cond :: Qbit \rightarrow (Bool \rightarrow U) \rightarrow U$$

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- An example conditional statement would be

$$ifQ :: Qbit \rightarrow U \rightarrow U$$
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- We can however "share" the state of one qubit with another.
- e.g. from a state $|\phi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ we can create the state $|\psi\rangle = \alpha\,|00\rangle + \beta\,|11\rangle$.

Multiple Qubits...

- $$
\begin{aligned}
&share :: Qbit \rightarrow QIO\ Qbit \\
&share\ qa = \mathbf{do}\ qb \leftarrow |0\rangle \\
&\qquad\qquad\qquad applyU\ (ifQ\ qa\ (unot\ qb)) \\
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- We can now use this to create the bell state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ from the $|+\rangle$ state.
- $$\begin{aligned} &bell :: QIO\ (Qbit, Qbit) \\ &bell = \mathbf{do}\ qa \leftarrow |+\rangle \\ &\qquad\qquad qb \leftarrow share\ qa \\ &\qquad\qquad return\ (qa, qb) \end{aligned}$$

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- In this example, the 2 qubits are Entangled . The state of one depends on the state of the other.
- In QIO, conditional statements are used to introduce entanglement into a computation.

Deutsch's Algorithm

- We now have everything we need to start defining quantum computations in Haskell. E.g. Deutsch's Algorithm

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- $$\begin{aligned}
&\mathit{deutsch} :: (\mathit{Bool} \rightarrow \mathit{Bool}) \rightarrow \mathit{QIO}\ \mathit{Bool} \\
&\mathit{deutsch}\ f = \mathbf{do}\ x \leftarrow |+\rangle \\
&\qquad\qquad\qquad\quad y \leftarrow |-\rangle \\
&\qquad\qquad\qquad\quad \mathit{applyU}\ (\mathit{cond}\ x\ (\lambda b \rightarrow \\
&\qquad\qquad\qquad\qquad\qquad\qquad \mathbf{if}\ f\ b\ \mathbf{then}\ \mathit{unot}\ y \\
&\qquad\qquad\qquad\qquad\qquad\qquad \mathbf{else}\ \bullet)) \\
&\qquad\qquad\qquad\quad \mathit{applyU}\ (\mathit{uhad}\ x) \\
&\qquad\qquad\qquad\quad \mathit{measQbit}\ x
\end{aligned}$$

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- So, we also provide three evaluation functions for QIO Programs
- $$run :: QIO\ a \rightarrow IO\ a$$
  $$sim :: QIO\ a \rightarrow Prob\ a$$
  $$runC :: QIO\ a \rightarrow a$$

Running Quantum Computations

- e.g.

$$> run\ (deutsch\ \neg)$$

$$True$$

$$> sim\ (deutsch\ id)$$

$$[(True, 1.0)]$$

$$> run\ (deutsch\ (\lambda x \rightarrow True))$$

$$False$$

$$> sim\ (deutsch\ (\lambda x \rightarrow False))$$

$$[(False, 1.0)]$$

$$> sim\ randomBool$$

$$[(True, 0.5), (False, 0.5)]$$

Quantum Data-types

- We don't always want to think of quantum computations simply as acting on qubits, for example, it would be natural to think of Shor's algorithm as having the type $shor :: Int \rightarrow QIO\ Int$.

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- We decided to implement a class of Quantum Data-types , that defines a relation between a classical type, and a corresponding quantum version of that type.
- $$\begin{aligned} &\textbf{class}\ Qdata\ a\ qa \mid a \rightarrow qa, qa \rightarrow a\ \textbf{where} \\ &\quad mkQ :: a \rightarrow QIO\ qa \\ &\quad measQ :: qa \rightarrow QIO\ a \\ &\quad condQ :: qa \rightarrow (a \rightarrow U) \rightarrow U \end{aligned}$$

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$$
\begin{array}{l}
\textbf{instance } Qdata\ a\ qa \Rightarrow Qdata\ [a]\ [qa]\ \textbf{where} \\
\quad mkQ\ n = sequence\ (map\ mkQ\ n) \\
\quad measQ\ qs = sequence\ (map\ measQ\ qs) \\
\quad condQ\ qs\ qsu = condQ'\ qs\ [\,] \\
\qquad \textbf{where } condQ'\ [\,] \qquad xs = qsu\ xs \\
\qquad\qquad condQ'\ (a : as)\ xs = condQ\ a\ (\lambda x \rightarrow condQ'\ as\ (xs +\!\!+ [x]))
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- and a Quantum Integer , which converts an Int to a (fixed-length) list of booleans, and defines a $QInt$ as a synonym for a list of qubits.

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- The paper goes into much more detail about the implementation of Shor's algorithm and the QIO evaluator.
- Shor's algorithm required that we have a set of unitary operators that can perform reversible arithmetic , specifically modular exponentiation.

More QIO Programs

- We have implemented other Quantum Algorithms using QIO, including Quantum Teleportation, and Shor's Algorithm.
- The paper goes into much more detail about the implementation of Shor's algorithm and the QIO evaluator.
- Shor's algorithm required that we have a set of unitary operators that can perform reversible arithmetic , specifically modular exponentiation.
- Many of the arithmetic functions require auxilliary qubits, so we have also added a unitary-let operation $ulet :: Bool \to (Qbit \to U) \to U$ that enables the system to keep track of them.

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- The Quantum Fourier Transform is essentially an implementation of the discrete Fourier transform, that can act on a quantum state, and is used in many quantum algorithms.
- It's use in Shor's algorithm is to find the order of a modular exponentiation function that's constructed depending on the input.

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- Trying to run the $notUnitary$ function will result in a run-time error.

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- It would also be possible to create a non-unitary single qubit rotation.
- The side-condition for rotations is that they must be unitary!
- Again, in both cases, failure to comply will result in a run-time error.

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- Thank you!